ABSTRACT

PERFORMANCE ESTIMATES OF A MEDIAN PLANE INJECTION SYSTEM FOR THE
MICHIGAN STATE UNIVERSITY CYCLOTRON*

By

John Dreisbach

Ions from an external source can enter the central region of a sector focused cyclotron by winding along a sector edge in the median plane. To use this method with the MSU cyclotron, the injected beam first travels along the inside of the dee stem to an "outer" electrostatic inflector located at a slightly greater radius than the last accelerated orbit. This outer inflector shifts the ions onto a cycloid-like trajectory, the drift of the cycloid carrying the particles inward, approximately following an iso-gauss contour at one of the hill-valley transitions of the sectored magnet. After approximately 35 cycles or "turns" of the cycloidal motion, the ions end up in the

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central region of the cyclotron where an "inner" electro-
static inflector system guides them onto a suitable normal
orbit.

In order to maximize turn separation at the inner
inflectors, the largest permissible injection energy should
be used. This is however limited by the voltage holding
capability of the inner inflectors. Injection at 3/1000 of
the final cyclotron energy is a compromise between these
factors and gives both adequate turn separation and reason-
able inflector voltages.

The focusing properties of such an injection system
were thoroughly investigated for the case of protons with a
final cyclotron energy of 42 MeV. The median plane (horiz-
ontal) acceptance of the system is about 3 times the mea-
sured emittance of the MSU cyclotron using the present in-
ternal proton source, after adjusting for the difference
in energy. The vertical acceptance of the injector, ne-
glecting the effects of nonlinear vertical forces, approx-
imately corresponds to the normal vertical emittance of
the cyclotron.
It was assumed that horizontal forces are independent of vertical position. Also, for convenience, the azimuthal magnetic field variation was described by only 12 Fourier coefficients; thus the field actually used corresponds to a cyclotron having a smoother flutter. Test cases estimate the effects of these various factors.

Since many external ion sources have low current outputs, a buncher is often employed with external injectors to increase the current out of the cyclotron. Detailed studies herein show however that a lack of isochronism in the injection path of this type of injector limits the improvement in transmission efficiency due to a sine wave buncher to a factor of around 3.5, for a 2% duty factor. The buncher causes a momentum spread of approximately 0.5% in the injected beam thus increasing the horizontal width of the beam at the cyclotron central region. This effect, when added to a horizontal distortion caused by the electric fields of the central electrode system, effectively increases the horizontal emittance by a factor of about 4 or 5.
One of the primary motivations for external injection systems is acceleration of polarized ions using some form of polarized ion source. So the influence of the cycloid-like motion on the polarization of protons was calculated: there is essentially no depolarization.

If a polarized ion source having a luminosity which seems feasible today is used with this injector, the estimated current out of the cyclotron should be around 50 nA. The radial acceptance of the proposed system is about a factor of 4 or 5 times smaller than that obtained with some existing injectors; and the axial acceptance, somewhat smaller. The energy spread of the beam from the cyclotron should be slightly better than elsewhere because the proposed duty factor is smaller. Finally the estimated increase in horizontal emittance, mentioned above, is similar in magnitude to that found elsewhere.
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1. Introduction

External ion source facilities and a beam injection system are frequently added to cyclotrons in order to obtain beams which are incompatible with the severe restrictions of internal sources. Most frequently the dominant interest is in obtaining polarized beams; in principle, performance should be improved for many other beams as well.

Four main types of injection systems have been utilized or proposed: 1) "Axial" injection systems use a hole bored in the magnet perpendicular to the median plane—the beam of ions is guided through this axial hole and then bent into the median plane by a 90° inflector or mirror (Po 65). 2) The "trochoidal" injection system uses an electric field in the median plane to cancel the magnetic force so that the injected beam follows a straight line from the outside to near the center of the machine (Be 67b). 3) "Stripping foil" injection systems direct a
beam of neutral or low charge ions from outside toward a foil near the machine center—when the ions pass through the foil charge exchange decreases their magnetic rigidity and, if the foil is properly positioned, the stripped ions are left on a normal internal orbit ready to be accelerated (Ah 69, Be 70a). 4) The "cycloidal" injection system uses the magnetic field gradient at a hill-valley boundary in a sector focused cyclotron—in such a field ions drift cycloidally along the sector edge from the outside of the cyclotron to the center, and then are shifted to a normal orbit by some form of inflector.

In order to choose between the various injection techniques, one would like to have detailed information on the performance characteristics of each type of system and also some idea of what equipment is required for each. This thesis has the purpose of providing such information for the case of a cycloidal injection system; the other types of systems listed above are already covered extensively in the literature and can be evaluated from those results.
Most of the injection systems for cyclotrons presently in operation are axial injectors. Cyclotrons with axial internal source holes can readily be adapted to this type of injector. The performance of axial injection systems is fairly good and is well verified. Probably the main deficiency is the potentially large increase in emittance as the beam comes out of the axial hole into the central magnetic field (Po 66a, Lu 68). Injection of ions in a low charge state and further stripping at the center of the cyclotron is usable only for heavy ions, obviously. If a neutral beam is injected, it is not necessary to use high voltage electrodes in the cyclotron but there are problems of obtaining a small diameter beam in the central region and of efficiently ionizing the beam at this location (Be 70a, Be 66b). The median plane method of injecting a beam radially in a straight line to the central region requires a system of electrodes which extends all the way from the outside of the machine to the central region and a large number of high voltage supplies (Be 67b). The cycloidal type injector has the potential for alleviating these deficiencies since there are no sudden magnetic
field variations that increase the beam emittance, there is no charge exchange after the ions leave the source, and no electric fields are required over most of the injection path.
2. General Description and Design Goals

A sector focused cyclotron satisfies the dual requirements of axial focusing and isochronism by means of azimuthal variation in the magnetic field. As shown in the simplified diagram (Figure 2.1), this type of magnetic field opens the possibility of using the cycloidal method of median plane injection in which the ions spiral along the interface between strong field regions called hills and weak field regions called valleys. This method was first proposed by V. Gladyshev and others at the Lebedeff Institute, Moscow, and was tested by this group in a model cyclotron (Gl 65). The Lebedeff experiment demonstrated that no electric fields were required over most of the injection path. Gladyshev's group also showed that the horizontal and vertical focusing properties were good, at least for the approximations of straight sector edges and suddenly changing magnetic field between the hill value and the valley value (assumptions which were reasonable for their experiment).
Fig. 2.1.—Schematic diagram of the cycloidal type injection.
For the MSU case, it is proposed to inject positive ions and, as will be indicated presently, an injection energy of 0.3% of the final energy appears best. With this energy and with the large magnet gap of the MSU cyclotron, the approximation of sharp hill-valley edges is much less valid and accurate knowledge of ion trajectories therefore requires numerical integration of the equations of motion.

Major features of the proposed injection system are shown in Figure 2.2. The maximum injection energy, which corresponds to acceleration across 155 kV, can readily be obtained by floating the entire ion source at this voltage, as is common in many types of accelerators. The following investigations hence assume that all the ion source equipment is on a high voltage platform.

After preacceleration the beam must be transported to the cyclotron magnet. Since RF electric fields could easily sweep the beam or increase its energy spread, it is proposed to lead the beam down the inside of the dee stem as shown in Figure 2.2.

Of several possible techniques for focusing the beam and providing shielding from the magnetic fringe field
Fig. 2.2.—Cyclotron cross section with injection system.
inside the dee stem, one which looks particularly attractive is a continuous, twisted magnetic quadruple. This consists of a long iron pipe with grooves in the inside containing conductors. The grooves are 90 degrees apart and rotate along the length of the pipe like the rifling in a gun barrel. The heliquad has been shown to have stronger focusing than comparable alternating gradient channels (Pe 70). The heliquad is described more fully in sections 3.2 and 4.2.

As is shown in Figure 2.2, the exit of the heliquad is about 48 inches from the center of the cyclotron since if it were nearer the iron in the pipe would be saturated by the fringe field of the cyclotron. Between the heliquad and the outer inflector, the beam must be guided along a straight path and be focused. Section 3.2 briefly describes a few devices that could be used in this region. The outer inflector, whose exit is located slightly beyond the radius of the last accelerated turn, bends the beam into the correct initial conditions at the beginning of the cycloidal path so that it will be correctly positioned relative to the inner inflector electrodes. The outer inflector is described in sections 3.3 and 4.3.
After about 35 turns of cycloidal motion inward along a sector ridge, the beam comes to the inner inflector system, a complex array of electrodes to focus the beam and steer it into a path that becomes tangent to the centered, accelerated orbit for its energy. Until this point, the beam remains shielded from the RF field by staying within the south dee. Soon after the beam becomes tangent to the centered orbit, it arrives at the edge of the dee and is then accelerated and extracted conventionally. Sections 3.4 and 4.5 describe the inner inflector system in detail.

The following design goals were considered in arriving at this injection system: The acceptance of the injector should be fairly large so that useful currents from low luminosity sources, such as polarized proton and deuteron sources, can be obtained. A related goal is that of minimizing all effects which would cause an increase in the effective emittance of the injected beam. Several such effects are discussed in section 4. Finally, if bunching is to be used, it is desirable that rays from different parts of the beam should have equal transit times from the source to the cyclotron center if the
energies are equal; i.e. the injection should be iso-
chronous. Section 3.1 deals with the buncher problem.
3. Construction

3.1 Ion source and buncher

The design of the injection system depends to some extent on the current available from the source. The most likely candidates for external sources usually deliver low current beams, viz. present day polarized sources provide nanoamps. to a few microamps. (Cl 68, Cl 70, Pi 69, Mi 68), and highly charged heavy ion sources also yield only several microamps. (Da 69, Kn 69). With these currents it is not necessary to consider the effects of space charge forces. And when a low current source is used on a cyclic accelerator, it is also clearly advantageous to include a "buncher" i.e. a device which accepts particles from the source over a long time interval and groups them into a small interval corresponding to the acceptance time of the accelerator cycle.

The simplest buncher consists of an insulated electrode through which the beam passes. The voltage on this
electrode varies with time so that an ion may experience a net change in energy as it sees an electric field first at the entrance and then at the exit. If the net energy gain for an ion reaching the electrode earlier is less than for a later one, then eventually the later one will catch up with the earlier one. In the ideal case all the ions within one RF period arrive simultaneously at the first accelerating gap of the cyclotron at the optimum instant. The simplest and cheapest (although not ideal) method is to use a sinusoidal buncher voltage. The frequency is that of the bursts coming out of the cyclotron, i.e., the dee frequency. In the present case, the energy change produced by the buncher will be very small compared to the injection energy, because of the large path length. Then if the voltage on the electrode is $V \cos wt$, the energy change is

$$dE = qV \left[-\cos\left(wt - \frac{lw}{2v}\right) + \cos\left(wt + \frac{lw}{2v}\right)\right],$$

where

- $V$ = peak electrode voltage,
- $w = 2\pi f$ (dee frequency),
- $t$ = time at which the ion passes the center of the electrode,
- $q$ = charge of the ion,
- $l$ = length of the electrode, and
- $v$ = velocity of the ion before the buncher.
The $v$ in this equation should actually be the velocity in the electrode, but since $dE$ is itself an incremental value the approximation holds. The first term is the energy change due to the gap at the entrance of the electrode and the second, that at the exit. This becomes

$$dE = -2qV \sin \frac{1}{2v} \sin \omega t.$$ 

Assuming isochronous injection, i.e., the total path length is the same for all rays, the time $T$ at which the ion arrives at the center of the cyclotron is

$$T = t + \frac{L}{v + dv} = t + \frac{L}{v} \left(1 - \frac{dv}{v}\right) \approx t + \frac{L}{v} \left(1 - \frac{dE}{2E}\right),$$

where

$L =$ total path length from the buncher to the cyclotron center and
$E =$ energy of the ion before the buncher.

The combination of the two preceding equations is

$$T = \frac{L}{v} + t + \frac{LqV}{E} \sin \frac{1}{2v} \sin \omega t. \quad 1.1$$

Ideally $T$ should be independent of $t$ over an RF period; but clearly this is not possible in the sinusoidal case. In this case, all the ions at the buncher in a certain
time interval centered about \( t = m \omega \) will be bunched into a considerably shorter time interval at the cyclotron central region. To decide what the length \( l \) of the buncher electrode should be, note that \( \sin \frac{lw}{2v} \) should not be too close to zero or else a very large voltage \( V \) will be required. This same problem was solved when the dee angle was chosen. It is only necessary to make the length of the buncher electrode equal to the path length in a dee at the injection energy. That is,

\[
l = \frac{hV}{\omega},
\]

where

\[ A = \text{angle subtended by the dee in radians, and} \]
\[ h = \text{harmonic no. of the acceleration mode}. \]

With a constant orbit geometry, \( l \) never changes.

In order to determine the only remaining unknown \( V \), use the fact that it is desirable to bunch as many ions into a time interval \( \Delta T \) as is possible with a single sine wave buncher. First differentiate equation 1.1 with respect to \( t \) to find the maximum and minimum.
\[
\frac{dT}{dt} = 1 + KV \cos wt,
\]

where

\[
K = \frac{LeV}{Ev} \sin \frac{1w}{2v'}
\]

1.2

Setting the derivative equal to zero yields

\[
wT = \cos \left( -1 \frac{1}{KV} \right)
\]

1.3

This equation in general has two solutions for \(0 < wt < 2\pi\).

Call the smaller one \(t_1\); then the other is \((2\pi - wt_1)/w\).

Equation 1.1 shows that \(t_1\) corresponds to the maximum \(T\).

(Choose the sign of \(V\) to make \(KV\) positive.) Next set the difference between the maximum and minimum of \(T\) equal to \(\Delta T\):

\[
t_1 - \frac{2\pi - wt_1}{w} + \frac{KV}{w} (\sin wt_1 - \sin (2\pi - wt_1)) = \Delta T.
\]

Simplifying this and solving for \(V\) yields

\[
V = \frac{2\pi - 2wt_1 + w_\Delta T}{2K \sin wt_1}
\]

1.4

where \(K\) is given in equation 1.2, and \(t_1\) is the smallest positive solution of equation 1.3 (which is to be solved
simultaneously with equation 1.4 to get $V$). The time interval at the buncher which corresponds to $\Delta T$ is obtained from equation 1.1.

As examples, these equations indicate that a two-gap sine wave buncher would compress the ions from a time interval of 99 RF degrees into 3.3°; 137° into 9.0°; 164° into 15.7°; and 201° into 31.7°. The corresponding values for $KV$ are 1.1, 1.2, 1.3, and 1.5, respectively. Note: because of the lack of isochronism of the proposed injection system, the actual bunching efficiency will be lower than the above numbers indicate. Under typical operating conditions (section 4.1), a peak buncher voltage equal to 0.3% of the ion source potential would pack 7% of the current from the source into a 7.2° cyclotron capture time.

3.2 Beam transport

The heliquad must act as a magnetic shield for the cyclotron fringe field and have a sufficient acceptance to transport the desired beam. Preferably, it should also fit in the existing dee stem and have reasonable power
requirements. A possible design, which would be usable with the 1.75 inch dee stem aperture, is shown in Figure 3.1. Typical parameters for this design include a 1.5 cm diameter clear aperture and a maximum field gradient of 1500 gauss/cm. The adequacy of these values will be demonstrated in section 4.2. Square wires 1/4-inch on a side are located 1.1 cm from the heliquad center. Replacing the wires with filaments at their centers, and ignoring all iron, the required field is produced with 2.67 kA per wire. If the average magnetic field line is actually within iron for about 2/3 of its total path length, the iron-free current should be divided by 3 as a first, rough approximation. Since about one half of the wire (OFHC copper) is water hole, the resistance comes to \(0.96 \times 10^{-3}\) ohm/m. A 2-meter heliquad would require a total power of around 6 kW, maximum, for the numbers given.

Near the cyclotron, the fringe field becomes so large that the iron in the heliquad would saturate. Consider a uniform magnetic field in the x-direction. Then the magnetic scalar potential is
Fig. 3.1.--Heliquad cross section.
\[ \phi = -H_0 \, r \cos \theta, \]

where \( x = r \cos \theta \).

This field will then be distorted when an infinitely long iron pipe of permeability \( \mu_2 \) is placed concentric with the z-axis. Let the outer radius of the pipe be \( a \) and the inner radius \( b \). A straightforward solution of this magneto-statics problem shows that if the permeability of the material outside the pipe and in the bore of the pipe is \( \mu_1 \), then in the bore

\[ \phi_b = -H_0 \left[ 1 - \frac{1}{\mu_2 + \mu_1} \left( \frac{b}{a} \right)^2 \right] r \cos \theta. \]

Cf. problem 13 of Chapter IV of (St 41).

Meanwhile, in the iron

\[ \phi_i = (cr + d/r) \cos \theta, \]

where

\[ c = \frac{2a^2 \mu_1 H_0 (\mu_2 + \mu_1)}{b^2 (\mu_2 - \mu_1)^2 - a^2 (\mu_2 + \mu_1)^2} \quad \text{and} \quad \]

\[ d = \frac{b^2 \mu_2 \mu_1}{\mu_2 + \mu_1}. \]
In Figure 3.1, \( b/a \) is about 0.65, not counting the pole tips. If it is desired to limit the field in the iron to 10 kG, and taking \( \mu_{2}/\mu_{1} \) equal to 1000, the allowed external field is 1.45 kG; while in the bore, a field of 10 G results. At maximum excitation of the cyclotron magnet, the fringe field falls to 1.45 kG at approximately 48 inches from the center of the cyclotron. Note that since the calculation assumed an infinitely long pipe, the magnetic field in the heliquad near the end will be somewhat larger. (It is assumed that any perturbation of the internal cyclotron field can be compensated by the "bump coils" in the valleys which are capable of producing a first harmonic of up to about 15 G at the outer turns at any azimuth.)

Between the exit of the heliquad and the entrance of the outer inflector, the beam is to travel in a straight line for a distance of 23 inches. Over this distance focusing elements should match the emittance of the beam at the heliquad exit to the acceptance of the outer inflector and following portions of the injector, and "bending" elements should provide horizontal forces to balance the forces arising from the cyclotron magnet fringe field. The maximum
magnetic field in this area occurs at the outer inflector entrance and reaches about 12.5 kG at the highest cyclotron magnet excitation. A system of electrostatic inflectors weaker near the heliquad and stronger near the outer inflector could be used to balance the magnetic fringe field. To obtain focusing, an electric field gradient transverse to the beam would be desirable, but an electrode configuration similar to the one at Saclay (Be 67b) would give very large fields at the electrodes leading to sparking. (The injection energy is 5 keV at Saclay.) Weak focusing could also be obtained by creating a magnetic field gradient. By using air core coils to generate a field gradient but not change the total number of flux lines cutting the median plane, the first harmonic contribution in the circulating beam region could be held to a minimum. If it is necessary to use strong focusing, it may be possible to use a heliquad in this region with windings on the outside of the pipe to shield it from the fringe field and to prevent distortion of the field in the circulating beam region, as is a standard technique (Hu 65) in the design of extraction channels for cyclotrons. If a design of this
type is used, the dee stem would probably have to be enlarged to contain the heliquad and windings.

Whatever device or combination of devices are used, it will be assumed herein that the emittance at the heliquad exit (which can be adjusted somewhat as discussed in section 4.2) can be transformed linearly to match the acceptance at the outer inflector entrance. The depolarization in a heliquad is treated in section 4.2; the depolarization inside a weak focusing channel would probably be less than in a quadrupole lens and thus negligible.

3.3 Outer Inflector

The construction of the outer inflector is illustrated schematically in Figure 3.2. (Its position in the cyclotron was shown in Figure 2.2.) The trajectory calculations assumed that the electric field of the outer inflector is simply the same as the field of a cylindrical condenser (no electric field component out of the plane of the drawing). Also the field is assumed to end abruptly at the ends of the deflector (no fringe field). Since
Fig. 3.2.--Outer inflector.
the height of the electrodes would be almost an inch and their separation 0.235 inch, these assumptions are reasonable.

Since all calculations were performed at the same main magnet excitation, it is not known whether the other magnetic fields drop so fast with radius that it would be necessary to make the inflector in two sections, with the outer section having a lower strength. In the case checked, the radius of curvature of the beam didn't change too drastically in the distance covered by the inflector. An outer inflector consisting of a polished stainless steel high voltage electrode and a tungsten ground plane should reliably hold the 60 kV (i.e. 98.8 kV/cm) necessary for 52 MeV proton operation. As a comparison, the electrostatic extraction deflector of the MSU cyclotron which is fabricated in this way achieves 137 kV/cm at a voltage of 96 kV.

After the beam exits from the outer inflector, it begins its cycloid-like drift toward the central region of the cyclotron. Figure 3.2 shows the first few loops of this motion.
3.4 Inner inflector system

After completing their looping drift inward along the hill-valley edge, the ions must be shifted onto a normal (i.e., eventually centered) orbit before leaving the dee so that they will thereafter accelerate conventionally. In contrast to the outer inflector, most of the electrodes in the inner system must not intercept the median plane: since the accelerating beam of the MSU cyclotron is 1 cm high or less, the inner inflector electrodes are positioned so as to leave a 1 cm clear aperture.

In order to obtain an electrode structure whose field could be conveniently computed, a "point charge" algorithm was employed for the design calculations. As the first step in this calculation, the field due to a point charge inside the dee was calculated (using a large number of image charges). Second, "electrodes" were synthesized by rows of point charges placed 0.36 inches from the median plane. Finally, equipotential surfaces of these point charge distributions were chosen to be the real electrodes and the strengths of the point charges were adjusted based on (a) maximizing the electric field in the median
plane, (b) minimizing sparking by maintaining low field strengths at the electrodes, (c) keeping the 1 cm vertical clear aperture, and (d) striving for electrodes with simple geometrical shapes, eg. curved right circular cylinders with hemispherical caps on the ends. Because of interaction among the electrodes and the several requirements they must fulfill, they differ significantly from these simple geometries; but the exact shapes are known and could be duplicated, albeit with rather difficult fabrication techniques. Alternatively, an electrolytic tank study could determine the fields of the simple geometries, the focal properties then being checked by numerical ray tracing.

The location of the 15 electrodes and the ground planes is shown in Figure 3.3, while Figure 3.4 shows several typical cross sections of the inflectors. (The small negative electrodes were omitted from the equipotentials of Figure 3.4 A-A, clarifying the drawing.) All the electrodes marked with minus signs in Figure 3.3 are at a potential of -48 kV in the case for which the calculations were made, viz.: 42 MeV protons. Since this is also
Fig. 3.3.--Inner inflector system
--median plane view.
Fig. 3.4.—Cross sections of Figure 3.3.
the voltage of the outer inflector, only one power supply and one lead-in are required for the negative high voltage. Although the electrodes marked with plus signs are at a potential of +2 kV, they could be enlarged and operated at a lower voltage, or vice versa; the thinner electrodes closely approximate curved circular cylinders, implying easier fabrication.

As the ions drift toward the center of the machine they are first influenced by the electrode pair D1 (Figure 3.3), one of three such separate or "dummy" electrode pairs. The dummy electrodes are necessary to obtain good vertical focusing properties: without them, the combination of the 1 cm vertical gap of the inner inflector system and the rather strong vertical beam blowup gave an unusably small acceptance. Independently adjustable voltages for D2 and D3, and the other positive electrodes as well, would provide for fine adjustment of the vertical focusing and of steering.

After about 5 turns in the inner inflector system, the ions arrive at the slit (Figure 3.3). If they hit one of the jaws, a small change in the injection energy will
get them through the slit. Monitoring the current on the slit jaws would give an indication of the horizontal width of the beam at this location and would signal possible slow changes in many of the injector components.

Once through the slit, the ions enter an inflector that bends their trajectory into a tighter curve. Ground plane G1, which constitutes part of this inflector and shields the injected ions from RF fields, has two holes in it to allow the accelerating beam to pass through. Although the median plane configuration of G1 is shown in Figure 3.3, this ground "plane" becomes wiggley near the adjoining positive electrodes (Figure 3.4 C-C), again causing fabrication difficulties.

The beam next experiences electric fields which oppose the magnetic forces. These electric fields are provided first by the "open" inflector whose cross section is shown in Figure 3.4 A-A, and then by the "final" inflector consisting of ground plane G2 and a high voltage electrode which intercepts the median plane. In construction the final inflector is very similar to the outer inflector described in section 3.3; but with a length of 1.1 inches,
a gap of 0.2 inch, and a radius of curvature of 4.4 inches. Fine voltage adjustments of the various positive electrodes could be used to guide the beam through the final inflector, the adjustments being performed while observing the first few accelerated turns with a beam probe. The presence of the final inflector will effect the electric field in the adjacent open inflector. Other factors that will cause small discrepancies between the electric fields used in the calculations and those of the actual physical device are the electrical connections to the electrodes and the two holes in the ground plane G1. Also, the high voltage lead-ins will have to be kept away from the beam or else electrostatically shielded so they will not perturb the ion trajectories.

The question of whether the proposed inflectors would hold the required high voltage (60 kv for 52 MeV proton operation) was investigated experimentally. The MSU cyclotron lab, mechanical shop constructed a mockup of the upstream end of the second inner inflector (ie. with a cross section like that in Figure 3.4 B-B). This mockup held -56 kv without excessive sparking. The high
voltage electrodes were stainless steel, and the ground plane and spark plates (above and below the pair of high voltage electrodes) were tungsten. After the test a 1 mm long copper shaving was found stuck on an electrode at a point of high electric field. Probably the shaving fell from the ground plane support, got attracted to the high voltage and became welded to the electrode with the subsequent spark. Most of the sparking occurred at or near this shaving. The maximum electric fields of the mockup are very nearly equal to those of the actual proposed inflectors at the same voltage, thus the inflectors will likely hold the required voltage, assuming the absence of copper shavings.
4. Performance

4.1 Ion source and buncher

The ion source performance determines to a large extent the performance desired from the rest of the injection system. In section 3.1 several types of ion sources were listed with maximum currents provided. Other parameters of interest are the emittance, energy spread, and if pulsed, duty factor and repetition rate.

Almost all known sources for external injection provide axially symmetric beams so the emittance is specified by one number. The acceptance of the cyclotron as well as the proposed injection system, on the other hand, is larger in the vertical direction than in the horizontal direction. Also, since the orbit geometry is fixed, the acceptances of the injector and the cyclotron are independent of the type of particles accelerated and their energy. The transverse momentum of the particles from an ion source tends to be fixed (often determined by the
plasma temperature) independent of the extraction voltage; thus the emittance from the source would be less at higher energies. As a result, if the source emittance were larger than the injector acceptance, the transmission efficiency would depend on energy. Since during acceleration in the cyclotron there is no mixing of longitudinal with transverse phase space and very little nonlinear distortion (Bl 69), and since the energy coming out of the cyclotron is 336 times the injection energy, the emittance at extraction is 18.4 times smaller than at the end of the injection process. (Actually this ratio is 1% larger for protons than for heavy ions due to the relativistic effect, ignored here.) A notable lack of uniformity exists regarding the units in which emittance is specified. One possible convention is to specify the emittance in units of mm mradians referred to 42 MeV. This convention will be used here. If the emittance needs to be given at the injection energy, for example, it will usually also be converted to the 42 MeV value.

For polarized ion sources, a good criterion of the quality of the beam is the product of the luminosity and
the square of the polarization. (The information to compute this product is not always complete in the literature on ion sources.) One of the best sets of figures yet quoted is the design data of the polarized source used at the Berkeley 88 inch cyclotron (Lu 69). The current expected is up to 10 μA; the 42 MeV emittance, 8 mm Mr; the proton polarization, 1; the deuteron vector polarization, 2/3; and the deuteron tensor polarization, 1; with the following definitions:

vector polarization, \( P_z = (N_+ - N_-)/\Sigma N \),

tensor polarization, \( P_{zz} = 1 - 3N_0/\Sigma N \),

where \( \Sigma N = N_+ + N_- + N_0 \); \( N_0 = 0 \) for protons, and \( N_+ \), \( N_- \), and \( N_0 \) are the occupation numbers of the corresponding states.

At Erlangen, Germany, a 70% polarized proton beam of less than 5.1 mm Mr. emittance with a current of 1 μA was achieved (Cl 70). The polarized source at Saclay yields 3 to 5 μA of protons. The emittance is not specified but assuming that the acceptance of the injection system is the same as an earlier mockup (not an entirely safe
assumption), the 42 MeV emittance comes to around 2 mm mr; the polarization at extraction is as much as 90% (Be 67h, Be 66b). A laser-pumped polarized $^{3}\text{He}^+$ source has an emittance of 1.5 mm mr at 42 MeV with a 4$\mu$A current and 5% polarization (Fi 69). A polarized lithium and cesium source has achieved a 20% polarized Li$^{++}$ beam at a current of 1.3 na (no emittance data) (Mi 68).

With one exception, the articles on heavy ion sources usually do not specify the emittances. The "ordinary" duoplasmatron or Phillips Ionization Gauge or Penning discharge types would probably have emittance at least as small as the polarized sources, based on the performance of unpolarized proton sources of these types (Wr 68, Be 67a). Of course many factors influence the emittance for a given general type of source, most especially the extraction aperture size. The types of ions and the currents available were listed in section 3.1. The only heavy ion source with an emittance estimate is the proposed "HIPAC" source (Da 69). It gives 32$\mu$A of $^{88}\text{Kr}^{20+}$ in an emittance area of 420 mm mr at 42 MeV; however, in this case the emittance should be adjusted to the actual cyclotron extraction
energy: the MSU cyclotron could accelerate such ions to over 250 MeV, at which energy the emittance is 170 mm mr. The HIPAC is also unique in that it is the only type of source with a nonnegligible energy spread, as a matter of fact, it is around half the injection energy. All the other sources listed characteristically have very small energy spreads, e.g. the energy spread of polarized sources comes mainly from the ionizer and is 20 to 50 eV, typically (Gl 66). The energy homogeneity restriction for the injector system is examined in section 4.4; the acceptance, in section 4.5.

In order to be able to bunch the injected beam, it is required that the energy spread before the buncher be considerably less than the energy that can be gained in the buncher, which is ±0.6% of the injection energy, roughly. The exact value depends on the usable capture interval of the cyclotron acceleration process and whether the injection is isochronous. Since the dee voltage of the MSU cyclotron has sinusoidal time dependence, the energy spread during acceleration is $1 - \cos (\omega \Delta t/2)$, assuming isochronism, where $\omega$ is the angular dee voltage frequency and $\Delta t$ is the
length in time of a beam pulse, if the starting phase is adjusted for maximum energy gain per turn (thus minimum energy spread). In order to achieve high extraction efficiencies, it is necessary to limit the total energy spread to less than the reciprocal of the number of turns. For a 200-turn geometry this restricts the capture interval to 11 RF degrees. Actually, because of finite electrostatic extraction deflector thickness, RF voltage ripple and other factors, the phase width must be considerably less than this for complete extraction. The anisochronism of the injection system amounts to much more than this, viz. even assuming perfect bunching, the time spread among different rays at the center of the cyclotron would be some 40 RF degrees, unless the emittance of the source were noticeably less than 2 mm mr, referred to 42 MeV. (Acceleration on the first harmonic is assumed; there is even less isochronism at the higher harmonics.) The optimum buncher voltage actually depends on the distribution of current within this time interval. A reasonably fair estimate is that 3/4 of the beam is spread over 20 RF degrees. Take a buncher voltage such that with isochronous injection the time
spread at the cyclotron center would be 9 RF degrees. This means that $V = 1.20/K$, and a time spread of 137 deg. at the buncher is packed into 9 deg. at the cyclotron center.

(See section 3.1 for notation and equations.) Adding the time spread of the injection process means that actually the 137 deg. at the buncher go into 29 RF deg. Now if the extraction efficiency is nearly 100% for as much as 7.2 RF degrees, 7% of the beam lies in that time interval, vs. 2% for the case of no bunching. The cyclotron energy spread for this case is 0.2%. If a smaller extracted energy spread is required the phase width must be reduced with a proportional loss of current (changing the buncher voltage won't help much because of the lack of isochronism in the injector). Although the extraction efficiency becomes lower, a net gain in current can be achieved by a large increase in RF phase width, again with a larger energy spread. The remaining 7% of the beam (for a 0.2% energy spread and a sine wave buncher) will be further diminished by effects to be described later.
4.2 Beam transport

There are no restrictions on the beam transport line between the ion source and the cyclotron vacuum box. The beam envelope may be as large as desired, many different types and sizes of focusing elements can be used as well as auxiliary devices such as energy analyzers, mass spectrometers, polarization rotators, and steering elements. Once inside the dee stem the previously described helical magnetic quadrupole channel will be used. With such a "heliquad," the axial component of the magnetic field can be ignored if the ratio of the period of one complete rotation of the pole tips to the maximum radius of the beam is greater than 20 (Pe 70). In this instance, G. Salardi has written the equations of motion and their solutions (Sa 68). If the aperture of the heliquad has a radius of 7.5 mm and the length of the rotation period is 30 cm, this ratio will be 40 and the results of G. Salardi should apply very accurately. By choosing a magnetic field gradient of 1342 gauss/cm for 125 keV protons (the value scales as momentum over charge), the motion is periodic along the length of the heliquad
channel. The periodicity of the ion beam is 47.5 cm in this case. If the total length is made an integer multiple of this value, the beam at the exit will have the same emittance shape as at the entrance (assuming a perfect quadrupole field in the heliquad). If all the focusing elements between the ion source and the heliquad are axially symmetric, e.g. Einzel lenses, then the beam at the entrance of the heliquad will be axially symmetric also since the emittance from most sources is. If these axially symmetric focusing elements bring the beam to a focus at the entrance of the heliquad, a 42 MeV emittance area of 6.8 mm·mr will be completely transmitted to the exit. To attain this acceptance, the proton beam should have a radius of 1.19 mm and a divergence of 33.1 mr at 125 keV at the heliquad entrance. It should be noted that the emittance of an axially unsymmetric beam can be larger. For example, if the heliquad is focusing in the Y direction at the entrance; the width of the beam in this direction could be 15 mm total and the divergence same as before, again assuming a focus at the entrance. However the area of 6.8 mm·mr is close to the maximum vertical acceptance
of the rest of the injection system and much larger than the horizontal one. Also, the acceptance of the heliquad could be made even larger by using a tighter spiral of the pole pieces. Since the acceptances of the injection system and the cyclotron, and indeed most nuclear experiments, differ in the vertical and horizontal directions, it might seem desirable to remove a little horizontal phase space area and place it into the vertical one. The heliquad can exhibit strong coupling in the two transverse directions (Oh 69) so it could be useful in this application. Such a scheme would of course be profitable only when the emittance from the ion source (assumed axially symmetrical) was in between the vertical and horizontal acceptances.

The focal properties of a heliquad can be approximated by an alternating gradient channel of quadrupoles having the same field gradient as the heliquad, no drift spaces and containing four quadrupoles per turn of the heliquad pole piece. The alternating gradient channel has a slightly larger focal length. D. Werren has examined the problem of depolarization due to magnetic quadrupoles (We 68). Specifically, he considered a beam tube radius
of 2.5 cm, two 13 cm long quadrupoles separated by 13 cm with a resultant focal length of around 15 cm, and found a depolarization for the maximally divergent rays of a few parts per thousand. Of course the depolarization is zero for paraxial rays. The result is energy independent. It might thus be expected that the heliquad channel would also have an acceptably low depolarization.

As was shown in section 3.2, the heliquad must end when the stray cyclotron field reaches 1.45 kG. This is 23 inches from the entrance of the outer inflector. Over this length, the magnetic field increases from nearly nothing at the exit of the heliquad to 80-90% of the field at the cyclotron center. Section 3.2 also describes possible devices for use between the heliquad and the outer inflector. If weak focusing is used, horizontal focusing can be increased and vertical decreased or vice-versa. It will be assumed that the emittance from the heliquad (which is adjustable within limits, as previously indicated) can thus be matched to the acceptance of the rest of the system, which will be described in the next sections.
4.3 Outer inflector

The array of focusing elements between the heliquad and the outer inflector just balances the magnetic field so that the central ray trajectory is a straight line. The outer inflector then bends the beam with a radius of curvature of 6 inches against the magnetic force. This inflector ends about 1/2 inch beyond the calculated position of the last turn of the normal internal beam of the cyclotron. In order to maximize overall vertical acceptance, the height of this outer inflector is the largest which will allow sufficient breakdown voltage in the 1.75-inch dee aperture. Calculations herein assume an electric field which is effectively uniform over a total height of 0.8 inch. The horizontal gap of the outer inflector was chosen to allow the high voltage electrode to operate at $-48\,\text{kV}$, the same as the inner inflectors, and the other electrode grounded to the dee; giving a gap of 0.235 inch. The median plane family of rays which determines the horizontal emittance actually requires a gap of 0.273 inch and electrode voltages of $-53.5\,\text{kV}$ and $+2.26\,\text{kV}$. Another family of rays which would fit in the .235-inch gap and still give a
similar emittance probably exists, but finding such rays is a lengthy task. The next sections give the details of the calculations.

There is an observable deviation from isochronism between the ends of the outer inflector. It is a small effect, as would be expected from the short time the ions spend in the inflector, and seems to partially compensate the larger lack of isochronism arising from other sources.

The spin axis of polarized particles should be aligned vertically along the field lines in the cyclotron, if possible. They should emerge from the heliquad aligned this way since they immediately experience a magnetic field which is nearly vertical for all rays. The electric fields before and within the outer inflector have no direct effect on the spin, so any depolarization must be determined by the small horizontal magnetic field components. The problem is similar to evaluation of depolarization during extraction, except that the velocities involved are some 18 times greater during extraction. G. Budyansky has calculated a depolarization that is many times less than .03 for practical cases during extraction (Bu 60); also there
are experimental results which show no detectable depolarization over the entire acceleration-extraction process in cyclotrons (Oh 70, Po 66b, Be 66b). Horizontal magnetic fields of the same order of magnitude are encountered as the ions drift down the sector edge, and no significant depolarization occurs there (see section 4.4).

4.4 Drift along the sector edge

The magnetic field of the MSU cyclotron was mapped at several main magnet currents using a grid consisting of 1 inch increments from R = 0 to R = 46 inches and 4 degree increments from θ = 4 to θ = 360 degrees, inclusive. The azimuthal variation was Fourier analyzed into cosine ("H") and sine ("G") components in this way:

\[ B(R, \theta) = B_0(R) + \sum_{n=1}^{N} \left[ H_{3n}(R) \cos 3n\theta + G_{3n}(R) \sin 3n\theta \right], \]

where N will be called simply "the number of harmonics" (Be 66). Since the general orbit codes available previously were not well suited for tracking ions as they loop
along the magnet ridge line, D. Johnson, of this lab, wrote a new computer code to integrate the equations of motion in Cartesian coordinates. He wrote versions of this code using Milne-Reynolds and Runge-Kutta integration methods and also a modified Runge-Kutta utilizing selected double precision integration constants. The modified Runge-Kutta version was used for most of the calculations herein, since it exhibited low accumulated errors and permitted convenient step size changes. The original versions included a term proportional to $z^2$ in the equation for $B_z$ and a term proportional to $\dot{z}$ in the equations for $\dot{x}$ and $\dot{y}$ (the dots indicating differentiation with respect to time). A test performed with and without these terms indicated that they were not necessary and thus the subsequent runs used only linear $z$-motion uncoupled from the median plane motion, as far as the magnetic field was concerned. A handy feature of the original code made it easy to change the number of harmonics used in calculating the azimuthal variation of the magnetic field. The original code was modified many times in the process of adding electric fields, the capability of simultaneously running two rays independent of
each other in $z$ (to reduce the required computer time), and polarization. As indicated previously, the electric fields of the open inflectors were obtained from rows of point charges. For the case in which only median plane motion is desired, the following procedure was used. First a separate code calculates the potential and field in the median plane due to a pair of charges each of unit strength located at $R = 0, Z = \pm 0.36$ inch. During integration of the trajectory, this data is then used to sum the contribution of each point charge at each step. (A total of about 300 point charge pairs were used to generate the fields due to all the open inflector and dummy electrodes.) For $z$-motion, the potentials are first calculated in the median plane and also slightly above the median plane. Then the orbit code uses these two sets of potentials to get the vertical electric field, assuming that it is proportional to $z$. The inner electrode fields are important at distances out to $r = 8$ inches, approximately. All of the programming is in FORTRAN IV and the time required for tracking one ray in the median plane (two different $z$ values) with depolarization determinations is
around 15 minutes on a Xerox Data Systems Sigma 7. Most of this time is spent in the electric field calculation just described. This method of calculation was used because it allows relatively easy modification of the electrode configuration. An analytic field is used in the closed inflectors, viz. the outer and final inflectors (see section 4.5) and very little computer time is required there.

As the ions drift along the hill-valley interface, they spend a relatively long time (2 microseconds) in regions of high field gradients. To verify that no important depolarization occurs, the direction of spin was integrated along with the other motions. Classically the equation of motion for a particle with angular momentum $\mathbf{s}$ and magnetic moment ($\mathbf{\tau s}$) is

$$\frac{d\mathbf{s}}{dt} = \mathbf{\tau s} \times \mathbf{B}.$$ 

Although the spin of a particle is a quantum mechanical effect, the only quantum mechanics needed for the depolarization calculation is a theorem which says that no quantum mechanics is needed. Usually called the correspondence
principle, it says that when averaged over many particle states (so as to become macroscopically measurable) quantum mechanical effects had better agree with classical physics. If the equation
\[ \hat{\mathbf{P}} = \langle \hat{\mathbf{J}} \rangle / jh, \]
where \( \hat{\mathbf{J}} \) is the angular momentum operator, defines the spin orientation vector; then the equation of motion
\[ \frac{d\hat{\mathbf{P}}}{dt} = -\Gamma \hat{\mathbf{H}} \times \hat{\mathbf{P}} \]
holds for all values of \( j \), where \( \Gamma \) is the gyromagnetic ratio. U. Fano gives this equation in (Fa 57) and also gives references to complicated quantum mechanical derivations.

The depolarization test actually performed consisted of running several median plane rays each with various initial conditions in the vertical direction. The initial polarization was \( P_z = 0.9899, P_x = 0.1, P_y = -0.1 \). (The beam would be polarized vertically initially ideally; the nonzero median plane components simulate a partial depolarization before the exit of the outer inflector.) The depolarization of protons, investigated in all portions of
the injection system downstream of the outer inflector, is roughly proportional to the maximum vertical displacement attained. The same magnetic field was used for the spin and position coordinates, i.e., the x and y components taken to be proportional to \( z \), and the z component, independent of \( z \). Out of a total of 14 different rays whose spin was tracked, the maximum change in \( P_z \) was 1%. The vertical initial conditions were chosen such that each ray attained the maximum \(|z|\) permitted by structure in the cyclotron. The injected ions used were 125 keV protons as in all computer runs; the number of harmonics used to calculate the magnetic field was 9 in five of the cases and 6 in the rest. Since the total depolarization is the average of all of the rays in the beam, including ones that remain near the median plane, it is completely negligible in this portion of the injection system.

The energy spread in the injected beam will broaden the beam horizontally near the center of the cyclotron. This is due to the spectrometer-type effect during the drift along the magnet ridge. The simplest possible approximation to the situation is a straight boundary line
separating a constant weak magnetic field and a constant strong one. Now consider ions starting at the boundary with an initial direction which is perpendicular to the boundary. After the first half-turn the distance from the starting point is proportional to the momentum of the ion. And on the second half-turn the trajectory is a different size semi-circle because of the sudden change in the field. The nonstartling but important point is that the distance to the starting point is still proportional to the momentum. In fact, even after 70 half-turns the distance to the starting point is still proportional to the momentum. The distance from the outer inflector to the inner inflector system is around 30 inches. The proposed buncher causes a peak to peak momentum spread of about 0.5% if the buncher is 4 m from the vacuum tank. (This is for first harmonic acceleration; the momentum spread is 0.25% for second harmonic, etc.) This means that the beam would sweep back and forth 0.15 inch at the cyclotron center due to the 0.5% momentum spread. This result is approximately confirmed by the actual integration of the equations of motion of the injected ions. The maximum energy spread
which can be tolerated is determined by this broadening of the beam. For instance, if the motion of the beam were perpendicular to the axis of the final (closed) inflector, the permissible sweeping would be equal to the inflector aperture minus the zero-energy-spread beam width; this difference is about 0.14 inch. Thus it might be desirable to increase the inflector path length so that the buncher wouldn't have to produce such a large momentum spread (the transport system is assumed to be isochronous). Locating the ion source system outside the shielding wall would also enable operators or technicians to manipulate it while the cyclotron is running. Even if all the beam fits into the final inflector, momentum spread causes an effective increase in the median-plane emittance. The inherent momentum spread of the ion source itself and of the preaccelerator limits how far the buncher voltage can be lowered by increasing the path length.

One problem which arose was how many harmonics must be used in calculating the azimuthal dependence of the magnetic field. (The conventional general orbit codes for the accelerating beam require 3 harmonics, i.e. up to
and including sine and cosine of 90.) The amount of depolarization for a given maximum vertical displacement varies only insignificantly between the 6 and 9 harmonic cases. The median plane motion differs by up to approximately .005 inch per loop (or turn). This is due to the fact that if the number of harmonics is 9, the maximum azimuthal gradients available become greater and the radius gain per loop increases with respect to the 6 harmonic case. The effect is evident only at the outer radii. The median plane error between the 6 harmonic calculation and the true motion in the exact magnetic field could probably be compensated for by a small change in the injection energy; thus it was decided to do the feasibility studies using 6 harmonics. But after completing most of the lengthy computer runs, it was discovered that the number of harmonics has a somewhat more important effect on the axial motion of the beam. To evaluate the dependence of the z motion on the number of harmonics used, several rays were run backward from the final closed inflector out to a radius of about 31 inches. For the same starting conditions, the ratios of the maximum axial displacement attained
in the 9 harmonic case to that in the 6 harmonic case were 5.2, 4.1, 3.7, 4.6, 1.7, 0.36, 3.8, and 1.5. Since the central geometry and especially the dummy electrodes were adjusted to give the best z focusing properties for 6 harmonics, it is not surprising that the beam tends to grow larger axially when the number of harmonics is 9. To get a very accurate picture of the z motion in the actual cyclotron magnetic field, it may be necessary to use even more than 9 harmonics. After finding out how many harmonics are required, the inner electrode structure would have to be changed to optimize the axial acceptance again. Since this would take a lot of computer time, only the results for 6 harmonics will be considered, corresponding to a field with a smoother azimuthal variation eg. one produced by a magnet with a larger gap. By optimizing the inner electrodes for more harmonics it may well be that the resulting axial acceptance would be as large as that found for the case investigated, since relatively small changes in the size of the dummy electrodes, for instance, cause a very marked change in the acceptance.
As with axial motion, it was observed that the median plane motion in the electric field free region was essentially linear for the maximum usable emittance area. The maximum nonlinearity is somewhere around 10% which, although noticeable, is not very important. The nonlinearities in the central inflector system, on the other hand, cause a large distortion of the median plane emittance. Both horizontal and vertical motions in this region are described in section 4.5. The overall performance of the injection system from the outer inflector to the final inflector is summarized in section 4.6.

4.5 Inner inflector system

Drawings of the inner inflector system were given in section 3.4. The method of calculating the electric field of the open inflector and dummy electrodes was briefly described in sections 3.4 and 4.4. The requirements of maintaining a degree of simplicity in the calculation, minimizing computer time, and ending up with a comprehensible set of results suggest that z motion
computations be limited to vertical forces which are proportional to \( z \) and horizontal forces which are independent of \( z \). This means that only two rays with independent vertical initial conditions are needed to specify completely the vertical motion; and the vertical and horizontal equations are separated (the vertical motion depends on the horizontal motion but not vice-versa). These are the same approximations used in the magnetic fields, for which case they are reasonable (due mainly to the fact that the magnet gap is large compared to the vertical size of the beam). In order to try to obtain a very rough idea of the errors involved in applying these approximations to the electric field of the inner inflector system, special computer runs were made. (It might be expected that nonlinear effects would be noticeable since the vertical clear aperture between electrodes is only 0.4 inch.) Most of the calculations used vertical electric fields obtained from potentials on the median plane and potentials on the plane at \( z = 0.05 \) inch, assuming that \( E_z \) is proportional to \( z \). If this were true, the same field would result using potentials from some other pair of \( z \) values. Computer runs
were made using vertical electric fields calculated from 
z equal to 0 and .05 and also 0 and .15 inch using rays 
with the same starting conditions. (Again the code tracked 
the ions backward from the final to outer inflectors.) 
The ratios of the maximum vertical displacement attained 
by the ions in the 0.15 inch case to that in the 0.05 inch 
case are 0.59, 1.3, 1.3, 2.2, and 1.5. The change in the 
magnitude of the axial acceptance was less than the ratios 
of individual rays for the cases tested. Of course, the 
shape of the acceptance areas will change.

The voltage on the electrodes of the open inflector 
was chosen to provide the correct radius of curvature for 
ions in the median plane. In the approximation used for 
the bulk of the computer runs, the same horizontal electric 
field components are used regardless of how far from the 
median plane the ions might be. Obviously this is not ex-
actly valid since an ion above (or below) the median plane 
will be closer to the electrodes and will be in the pres-
ence of a considerably stronger horizontal electric field. 
It is quite easy to get an upper limit on the result this 
effect would have by comparing the bending strength of the
open inflector configuration at the median plane vs. that away from the median plane. Since the field at 0.2 inch is as much as 30 to 35% higher than at the median plane, it can be seen that particles near $z = \pm 0.2$ inch in the open inflectors will all be lost, as an upper limit. Actually the runs with the horizontal force components independent of $z$ and linear vertical forces indicate that the beam envelope tends to be small vertically in the open inflectors and, since the vertical focusing is fairly strong here, any one ray is at large values of $z$ only for a short time. About the simplest calculation that would give some indication of the motion of the particles in the more complex field uses horizontal force which can assume two values depending on how far the particle is from the median plane. The calculation consisted of a series of computer runs all with the same horizontal initial conditions but with different vertical initial conditions. There were two sets of horizontal components of electric field: the standard ones—i.e. median plane values, and the non-median plane ones ($z = 0.12$ inch). The standard horizontal components were used whenever the particle was less
than 0.07 inch from the median plane and non-median plane ones were used otherwise. Thus the resulting x-y motion depends on the z motion. Also, although the vertical electric fields are just the standard linear ones obtained from the potentials at $z = 0$ and $z = 0.05$ inch, the actual resultant z motion is not linear since different rays follow different x-y paths yielding different vertical forces.

The computer code tracked the particles backwards from the exit of the final inflector using various initial conditions in the z direction with the restriction that each attain nearly the maximum displacement from the median plane permitted by the structure. The nonlinearity is exemplified by two rays starting on the median plane with a 16% difference in vertical divergence, otherwise identical; almost a fourfold change in the maximum vertical displacement occurred. The projections of the various trajectories onto the median plane create an envelope that is about .2 inch wide for much of the path length. At the outer inflector exit, the horizontal distances between the ray lying in the median plane and the other rays (measuring perpendicular to the median plane ray) are +.17, -.14, -.05, -.04, and -.25
inch. The turn separation at this point is about .25 inch. If the effect considered here causes a horizontal spread of around a quarter inch or more in addition to the inherent spread due to the incoming horizontal emittance, some of the beam will be lost. The lost rays will be those of large vertical displacements in the inner inflector system. The beam not lost will effectively reduce the horizontal acceptance of the injector by widening the beam. (It is little consolation, but even assuming no variation of the horizontal fields with z, the horizontal emittance gets fairly distorted, as you will see quite soon.) In the case tested the vertical acceptance was about 65% of the value obtained for the simpler case of horizontal forces independent of z. This number will shrink further because some of the rays stray too far horizontally and are lost. Actually, of course, all the results of this very artificial test run, using horizontal forces which change suddenly from the value correct at z = 0 to the value correct at z = 0.12 inch whenever the particle's vertical displacement exceeds 0.07 inch, are not meant to be interpreted with numerical exactness. They only give a rough estimate of the kinds of
errors involved in the linearized approximations which are necessary to carry out quantitative calculations. It should be emphasized that all of these nonlinear effects result from the electric fields. As said previously the nonlinear and emittance mixing capacities of the magnetic fields are negligibly small.

The closed inflectors are assumed to be large enough vertically so that $E_z$ is negligible inside in the region occupied by the beam which is .4 inch high in the final inflector and .8 inch in the outer inflector. At the exit of both closed inflectors the potential changes abruptly from the cylindrical capacitor distribution in the inflectors to zero outside. At the entrance to the final inflector the potential changes suddenly from that determined by the open inflector electrodes to the cylindrical capacitor form. The sudden potential change means a sudden change in the speed of the ion. A delta function electric field effectively exists at these points directed parallel to the inflector axis. Since the transverse velocity does not change, there is a sudden change in the particle direction. And since the potential in the closed
inflectors is more negative than outside, usually, the positive ions go faster inside and are more nearly parallel than outside the closed inflectors. This slightly unphysical delta function electric field is simply an easy way of approximating the fringe field of a real inflector which would extend over a measurable distance. This distance is so short in the case of an inflector with a horizontal gap of around 0.2 inch that the sudden approximation used is probably accurate.

To determine the acceptance area in the median plane and its shape, a series of rays were tracked from the final inflector to the outer inflector. The magnetic field was generated from 6 harmonics, creating an error of a few mills per loop which can be compensated by a small change of injection energy, as described previously. In the median plane the electric fields are quite accurate, limited only by any fabricational difficulties in the construction of the various electrodes and ground planes. The median plane motion is, however, nonlinear, necessitating a large number of rays in order to determine the acceptance accurately. The important objectives are
maximizing the acceptance area and maintaining a shape that is rather compact during the acceleration in the cyclotron to insure passage through various slits and apertures and provide both low divergence and small beam size on target. The median plane emittance of the beam as it emerges from the final inflector prior to crossing the first accelerating gap is shown in Figure 4.1. It meets the criterion of compactness while the lumpy appearance arises from an attempt to include as much area as is feasible. The area enclosed by the figure corresponds to 2.4 mm mr referred to 42 MeV (or 43.7 mm mr at 125 keV). The area possibly could be increased by bulging out the boundary at some places, but additional calculations would have to verify acceptable vertical focusing and horizontal position and direction at the outer inflector. Because of the substantial nonlinearities in the median plane caused by the electrodes of the inner inflector system, the acceptance area at the exit of the outer inflector is in bad shape, as shown in Figure 4.2 (the choice of the origin is arbitrary). Assuming that the emittance from the ion source is elliptical and the beam optics between it and
Fig. 4.1.--Median plane emittance at the exit of the final inflector.
Fig. 4.2. — Median plane emittance at the exit of the outer inflector.
the outer inflector exit are linear (both likely true, approximately), then these optics must be adjusted to produce the ellipse which coincides best with Figure 4.2. There are these three different cases: 1) The ion source emittance is a lot smaller than 2.4 mm mr. An ellipse of around 1 mm mr or less (again at 42 MeV) fits entirely within the shape of Figure 4.2 and thus would end up as a subset of Figure 4.1 with little dilution of the emittance area and no loss. Most sources considered for external injection don't have such small emittances. 2) The ion source emittance is around 2.4 mm mr referred to 42 MeV. In this case the emittance ellipse will overlap with the acceptance area of Figure 4.2; some of the beam will be lost and not all of the area of Figure 4.1 will be filled. For an ion source emittance of 2.4 mm mr, perhaps 1/3 of the median plane area will be lost; for twice that emittance, perhaps 2/3 would be lost. 3) When the source emittance is large compared to 2.4 mm mr at 42 MeV, the ellipse can be made to enclose the area shown in Figure 4.2 and 2.4 mm mr would be accepted by the injector regardless.
of what the emittance of the source might be; the rest would be lost.

The vertical acceptance, for several of the median plane rays of Figure 4.1 was determined making use of the approximations that all vertical forces are linear and that horizontal forces are independent of $z$. The number of harmonics used to determine the magnetic fields was 6. The following restrictions (required by structure in the machine) are placed on the vertical motion: The total height of the beam is limited to 0.8 inch in the outer inflector, 0.4 inch in the inner inflector system, and 1.6 inches in between. In addition, the half-angle of divergence immediately after the final inflector is limited to 25 mr. As in the median plane case, the vertical acceptances are also influenced very strongly by the electric fields of the inner inflector system. As mentioned previously, the dummy electrodes are needed only for improving the vertical focusing in this region. Much experimentation was required to obtain the vertical acceptance of 7.76 mm mr at 42 MeV for the best median plane ray tested. Most trial dummy electrode configurations gave very low vertical
acceptances for most of the ions having horizontal initial conditions different from the central ray. In this case, however, the vertical acceptance areas for rays chosen at random around the periphery of Figure 4.1 (or Figure 4.2) are 6.73, 4.76, 7.55, 7.23, 7.43, and 5.94 mm at 42 MeV. Since the vertical acceptances tend to be low for rays away from the center portion of the area shown in Figure 4.1, the average of all the horizontal rays would probably give a number fairly close to 7 mm at 42 MeV; the approximations used in these calculations were stated at the beginning of this paragraph. Figures 4.3 and 4.4 show the variety of vertical acceptances for different horizontal rays. The figures show only the positive z regions since the boundaries are symmetric about the origin. The areas shown are one half the maximum possible acceptances limited by structure in the cyclotron; areas read off the graphs must be divided by 18.4 to give the 42 MeV acceptances. (Since the shapes aren't ellipses it is not immediately obvious by looking at the drawings that the connecting transformation is linear; it is though.) The largest area shown (bounded by the solid lines in the drawings) is from
Fig. 4.3.--Vertical acceptances at the final inflector exit.
Fig. 4.4.--Vertical acceptances at the outer inflector exit. (Note scale change from Figure 4.3)
a ray in the central portion of Figure 4.1; the rest are all boundary points. From Figure 4.4 it is evident that if the emittance of the source is somewhere around 6 mm mr (42 MeV), some of the beam will be lost and some of the vertical acceptance area will not be filled. Again because the rays on the perimeter of the horizontal acceptance area tend to have smaller and more eccentric vertical acceptance areas associated with them, Figure 4.4 which shows the maximum variety of such areas, fails to indicate that actually the vertical acceptance areas coincide with each other fairly well for the majority of the horizontal rays. Thus it would be reasonable to assume that well over half of the vertical emittance area of a source with a 42 MeV emittance of 6 mm mr would be transmitted by the injection system, for example, with the approximations of these calculations.

Thus it has been demonstrated that the inner inflector system plays a most important role in determining the optical properties of the injected beam. The motion in the median plane becomes markedly nonlinear under the influence of the highly localized electric field from the
several electrodes. The vertical acceptance depends critically on the inner inflector system because of the strong vertical focusing and defocusing forces of the electric fields and also because the presence of the electrodes restricts the vertical aperture to about a centimeter. The usable vertical acceptance would be uselessly small without careful tailoring of the inner electrode configuration to yield reasonable vertical focusing properties. Section 4.6 gives operating characteristics of the overall injection system when used with typical external ion sources, and section 5 compares this performance with different types of injection systems and with internal ion sources.

4.6 Performance summary

Because of the low current output of many external ion sources (higher current sources usually can be modified for internal operation), an often used criterion of the quality of an injection system is the transmission efficiency. This is somewhat inadequate for two reasons. First, the transmission of a given injector depends
strongly on the properties of the ion source used. For instance, the beam quality of an unpolarized proton source is usually very high, resulting in a higher transmission efficiency than for polarized sources at the same currents. Secondly transmission alone provides no indication of the beam quality after extraction from the cyclotron; e.g., it may be possible to increase transmission by increasing the cyclotron RF phase width, but this might make the energy spread unacceptably large. Rather arbitrarily assuming that the energy spread should be 0.2%, total, the RF phase width is limited to 7.2 RF degrees, ignoring other causes of energy spread. This phase width permits complete extraction. Section 4.1 demonstrated that 7% of the beam could be bunched into this time interval with a single sine wave buncher for the case of first harmonic cyclotron operation. This is the most important source of loss of transmission efficiency for most ion sources; it is worse at higher harmonic acceleration; if the injection path were exactly isochronous, the efficiency would be about four times better (and no additional loss at higher harmonics).

The 7% efficiency applies to the case in which the 2.4 mm mr
horizontal emittance (42 MeV) is filled and assumes that rays with different vertical motion but the same median plane projection are isochronous with each other. A typical modern polarized ion source might have a current output of 5μa and an emittance of 5 mm mr at 42 MeV. Such an emittance would fill a large portion of the distorted acceptance area of Figure 4.2 with proper horizontal focusing and about 2/3 of the median plane beam would be lost. In this case the lack of isochronism would lead to the 7% time structure efficiency quoted earlier. The vertical acceptance is nearly adequate for the entire emittance, under the approximations of section 4.5. Assuming that the path length from the buncher to the central region is such that the lack of isochronism causes a horizontal emittance increase to around 7 mm mr at the cyclotron central region, about 1/2 of the beam within the 7.2 RF deg. phase width should come out of the cyclotron. In this case the overall transmission efficiency would be about 1%. If the source delivers about 5μa, an approximately 50 nA beam with a total energy spread of about 0.2%, and an effective vertical emittance of 6 to 7 mm mr
and a horizontal one of half that should be extracted from the cyclotron.

The increase in radial emittance is due to beam sweep during injection caused by the buncher energy spread. It is possible that the extracted energy spread will be increased a little during the acceleration process or that the horizontal emittance will effectively increase because of beam sweeping in the cyclotron fringe field or bending magnets due to the energy spread (see Bl 69), but such effects aren't directly related to injection systems. It can be seen that for sources with emittances greater than about 10 mm mr (42 MeV), the transmission is proportional to the reciprocal of the square of the emittance.

As an example of a source with a higher luminosity, on the other hand, consider the polarized $^3$He source (Ba 68 and Pi 69) that was described briefly in section 4.1 as having an emittance of 1.5 mm mr referred to 42 MeV and a current of 4 $\mu$A. Most of this emittance area would fit into the horizontal acceptance of the injector and in addition the deviation from isochronism would be much less since the "extremities" of Figure 4.2 would not be filled.
The more central portions of the area have more nearly equivalent transit times to the machine center. It appears that in the 1.5 mm mr case, the isochronism is better by at least a factor of 2, leading to considerably better bunching efficiencies. This combined with very small losses due to emittance matching should give a total transmission efficiency of 8% or an extracted current of 320 nA (same energy spread as before). The energy spread of the ion source (and preaccelerator) has been assumed to be small compared to that caused by the buncher, which is a good approximation for most ion sources.

The HIPAC ion source (Da 69), in sharp contrast, is expected to deliver a Kr\textsuperscript{20+} beam of 32 \(\mu\text{A}\) in an emittance area of 170 mm mr referred to 250 MeV with an energy spread of about 50% of the corresponding injection energy of 743 keV. Only about a 1% energy spread will be transmitted by the injection system. The buncher is useless here. The vertical and horizontal emittances being 7 and 2.4 mm mr (now referred to the present higher extraction energy) means that the extracted current would be 7.4 pA or two million ions per second; it is assumed that the
vacuum is good enough to prevent charge-changing collisions before or during acceleration.

R. Beurtey is planning a polarized ion source having an ionizer using the $\text{H}^+ + \text{H}_2^0 \rightarrow \text{H}_2^0 + \text{H}^+$ reaction in a very high magnetic field that is expected to deliver 0.4 mA of polarized protons in 3.1 mm mr (42 MeV) with an energy dispersion of $\pm$230 eV (Be 68) which will cause some loss of buncher efficiency. This high current raises a question that has been ignored until now, viz. what is the effect of the space charge forces? Especially with a buncher, the peak currents would be milliamperes, with such a source. This is the same magnitude of current encountered from the internal nonpolarized proton ion sources. In the present case, reliable calculations of the space charge limit of the injection system would be very time consuming and difficult, but it can be supposed that it would be somewhere near the range of space charge limits found in axial injection systems i.e. 0.3 to 0.8 mA, peak (Co 62, Cl 69).

The injection energy is higher in the present case but there are more severe restrictions on the size of the beam. At any rate, the polarized sources in operation now, and
also many of the heavy ion sources unsuitable for location inside the cyclotron, yield beam currents far below the space charge limit.

The polarization of the extracted beam should be essentially the same as that of the source, the only real possibility of depolarization is in the heli-quad, and there it is unlikely. The rest of the injection system has been shown to be free of serious depolarization effects (sections 4.2 through 4.5). The question of depolarization during the acceleration process in isochronous cyclotrons of less than about 100 MeV has been examined theoretically (Po 66b, Bu 60) and experimentally (Be 70b, Be 66b, Oh 70), and the amount of depolarization was found to be small in all cases.

External sources of highly charged heavy ions, particularly the one at Saclay (Kn 69) giving 0.5 to 1 μA of Li$^{3+}$, C$^{5+}$, O$^{6+}$, and Ne$^{7+}$, could be used with this injection system if the emittance were acceptable. Unfortunately, there is a lack of data on the emittance of such heavy ion sources, both internal and external. The fact they operate fairly successfully with conventional cyclotrons indicates
that the beam quality is not too bad. If it is similar to that of polarized ion sources, for example, the previous estimates of the transmission efficiencies and extracted beam quality apply.

The transmission efficiency could always be increased somewhat by lengthening the duty factor in the cyclotron, at the expense of reducing the extraction efficiency and increasing the energy spread. The exact transmission efficiency then depends on the thickness of the electrostatic deflector septum, how well its shape is matched to the shape of the last orbit, and the centering as well as the duty factor; making it difficult to give a numerical estimate. The MSU cyclotron normally operates with restricted RF phase widths and single turn extraction, so the performance estimates given have assumed these conditions.

The effect of the inner electrode system on the first few accelerated turns could probably be compensated by the centering coils. The energy of the beam after one turn is over 2.5 times the injection energy, reducing the influence of the stray electric fields. The injection
system at Saclay uses electric fields to counteract the magnetic field for the entire median plane injection path. The electrodes in this case have voltages of +20 kV and -23 kV (and other lower voltages) and the accelerated orbits are disturbed by less than 2 mm radially and negligibly vertically (Be 67b). It would be expected that the results in the case of the proposed MSU system would be similar, thus the beam could be recentered. Section 5 will compare the estimated performance of this proposed injection system with that of existing injectors and with internal sources.
5. Comparison

At present axial injection systems are the most popular way of getting ions to the center of a cyclotron from an external source. The system designed by Powell's group at Birmingham was the first reported operation of an injection system in an actual cyclotron (Cl 69b). In this system the ion source is located about two meters above the cyclotron median plane and 11 keV deuterons pass through a 2 inch diameter hole in the magnet pole focused by 6 einzel lenses onto an electrostatic mirror at the median plane. The aperture of the lenses is 2.2 cm diameter; and the hole is reduced to 1 inch in the pole tip. Various versions of the system are described in the literature (Co 62, Po 65, Po 66a, Po 66b, and Cl 69b). A special feature of this system is the presence of grids of vertical wires in the dees to improve vertical focusing. The grids also intercept 30% of the beam and distort the radial emittance. The gauze of the electrostatic mirror
also contributes to the distortion of the emittance and intercepts another 35% of the beam. All axial systems have the possibility of enlargement of the transverse emittance as the beam comes out of the pole tip hole into the central magnetic field. Rays with a transverse momentum component will experience a transverse force due to the longitudinal field and the total transverse momentum of these rays will increase while the longitudinal momentum decreases. The change in phase space area due to the varying electromagnetic momentum term is discussed by Powell (Po 66a). The effect is minimized by adjusting the focusing properties of the system so that the beam is at a narrow waist between the hole exit and the mirror. Also there is a distortion of the electric field of the einzel lenses due to electron trapping which approximately doubles the emittance. The net result of all these effects is that the 42 MeV emittance is 10 mm mr at the source and about 44 mm mr in the circulating beam. The expansion factor is thus similar to that in the MSU cycloidal system (horizontal), but for quite different reasons. The acceptance of the Birmingham injector is larger, around 20 mm mr (42 MeV). A sine wave
buncher is used and the total transmission efficiency is 2%. A large RF phase width is used during acceleration; in fact, the finite injection energy allows this to be up around 90 degrees. If the electrostatic mirror is placed at the exact center of the cyclotron, the injection energy is limited to about 1/5 of the dee voltage in order to obtain centered orbits. Also the voltage required on the mirror becomes impractically large for injection energies much greater than about 15 keV. In spite of these low energies, the space charge current limit can be fairly high: using an RF source, the Birmingham group injected about 250 μA to the mirror.

The axial injection system for the 88-inch cyclotron at Berkeley uses a polarized proton and deuteron source 4.5 meters above the median plane and a beam transport system composed of three sets of electric quadrupole triplets. A gridded electrostatic mirror placed on the axis in the median plane is similar to the one at Birmingham, except a finer grid mesh is used (Cl 69a). The source is expected to give up to 10 μA eventually and the measured emittance is 8 mm mr referred to 42 MeV (Lu 69). The
7.3 cm I.D. of the electric quadrupoles allow an acceptance of the injection line of 12.4 mm mr (42 MeV). The injection energy is 10-15 keV. The Berkeley group calculates that 0.6-0.8 mA can be transmitted if the 42 MeV emittance area of the source is no more than 3.1 mm mr and is shaped correctly (Re 69). Inside the axial hole through the magnet yoke and pole, it is usually necessary to take the magnetic field into account since it is usually around 1 kG at maximum magnet excitation. The Berkeley group made detailed studies of this region, especially the exit of the hole (Lu 68). At this point the decrease in longitudinal velocity was "less than 5 parts per thousand." Since energy does not change, the transverse velocity would have to increase to one-tenth of the longitudinal velocity, if there were a 5 part per thousand decrease in the longitudinal velocity; thus indicating again the importance of taking care to minimize the increase in emittance at the exit of the hole. In (Cl 69a), the following experimental data is listed: with a 12 keV polarized proton injection beam the total transmission efficiency was 3% using a sine wave buncher. (The current out of the source was 2 mA.)
A transmission efficiency of 1.5% was achieved without the buncher. In a test with a nonpolarized source, the efficiency was 4.5% without the buncher, indicating that the transmission may depend on the emittance of the source.

At the 60 MeV cyclotron of Grenoble an axial injection system is in operation which uses an electrostatic deflector instead of a mirror, in order to eliminate the beam losses on the grid. The design problems are somewhat more complicated but a slightly larger injection energy is feasible (19 keV for the highest cyclotron energy). With a nonpolarized source the transport optics allow 150 μA at 12 keV. The RF phase width is calculated to be 50 degrees (Pa 69). The total transmission with a polarized proton source is 0.4% (Cl 69b), or somewhat less than the other axial systems mentioned or the estimates of the MSU cycloidal method; this value was however obtained without a buncher.

The axial injector of the Cyclotron Corporation's 15 MeV H⁻ machine uses electrostatic quadrupole focusing elements and a gridded electric mirror. The maximum injection current and energy are 2.5 mA and 15 keV, demonstrating
the ability of this method to conquer space charge problems. The transmission efficiency with a buncher is 1.5% (Cl 69b).
The performance data of these examples are probably representative of the axial systems at more than a dozen institutes (Kh 67).

The second injection scheme to be used on a full scale cyclotron and the first to function with a polarized beam, was median plane injection of an atomic beam at thermal velocities (Cl 69b). In essence, a polarized ion source is modified by removing the ionizer and placing it at the center of the cyclotron. Since a neutral beam is used, there is no problem of the cyclotron fields disturbing the trajectory, but it is difficult to focus. In the fixed energy cyclotron at Saclay, this method was used to obtain a polarized deuteron beam of .03 nA on target with an energy spread of 1.5% (Be 66b, p. 80). This method was also used to inject polarized protons into the Saclay variable frequency cyclotron (in the case of protons it is necessary to use a liquid nitrogen cooled ionizer to reduce the unpolarized background). Up to 0.1 nA in the external beam was obtained with a polarization of 50 to 70%
resulting from contamination by residual gas in the center of the cyclotron (Be 66b, p. 82). It is unlikely that the efficiency of the thermal velocity atomic beam method will ever attain a value comparable to other injection schemes.

A somewhat more promising approach is to inject fast neutral particles as at Dubna and at Rez, Czechoslovakia (Be 70a, Be 70b). Polarized protons or deuterons are first produced conventionally then accelerated and focused; next electrons are added to the ions and the fast atomic beam enters the cyclotron radially, the atoms being stripped at the center. The injection energy of the Rez double charge exchange system is 45 keV and the beam is neutralized with about 40% efficiency in a 20-cm long chamber containing hydrogen at .01 torr. Minimum divergence of the neutral beam is required which implies a large beam diameter at the neutralizer, but small apertures on both sides of the neutralizer are desired to reduce $H_2$ flow and pumping requirements. Only 22% of the atomic beam hits the 1-cm diameter stripping foil near the center of the cyclotron, almost half of this is ionized. As much as 4.5 $\mu$A of polarized protons are available before the
neutralizer and the total transmission efficiency is 0.14% with a buncher. The injection system at Rez is still relatively new and it is thought that modifications can be made to increase the efficiency (Be 70a).

C. A. Tobias first proposed a method of injecting heavy ions at a low charge state into the median plane in an arc with a large radius of curvature and then stripping them to a higher charge state near the cyclotron center (To 51). Following his suggestion the group at Orsay is building an injector system using a heavy ion linac for preacceleration to 1.1-1.6 MeV per nucleon. The system wasn't yet operational as of 1969 (Ah 69).

The method presently in use at the Saclay variable frequency cyclotron consists of balancing the magnetic field with an electric field for the entire length of the radial median plane injection path. The system of electrodes uses 13 separately regulated voltages from -1 kV to +20 and -23 kV to compensate both the fringe and central magnetic fields and to shape the transverse electric gradient to provide vertical and transverse focusing (Be 67b). The injection energy for protons is 5.2 keV with an 18.4 KG central
field. Up to 5 µA of current is provided by the source. The polarization after extraction from the cyclotron is 90%. A total transmission of 1.4% has been attained (i.e. 70 nA external) (Cl 69b). The acceptance of the system now in use is not known, but an experimental mockup having 8 instead of 13 separately regulated voltages had a 42 MeV acceptance of around 0.33 mm mr (B6 66b, p. 84).

Cyclotrons with internal ion sources produce unsurpassed nonpolarized beams of light positive ions, at least up to lithium. Here there are problems since there are no room temperature gaseous compounds of lithium, the second and third ionization potentials are high, and lithium vapor is chemically active (Va 69). A satisfactory internal lithium source was developed at the Kurchatov Atomic Energy Institute in Moscow, even though the limited space available is a definite disadvantage, and the source lifetime is 40 hours. Since lithium contamination of the dees lowers the work function and causes sparking, the dees have to be washed every two weeks. The size of the extractor slit in the source is 1 mm x 2 mm and the external beam is 100 nA of 60 MeV Li$^{3+}$ ions, or 8 µA of 36 MeV Li$^{2+}$ (Va 69).
Lithium sources suitable for external injection systems seem to be absent from the literature. An exception is the source used at Saclay (Kn 69), but since it provides only 500 nA of Li$^{3+}$, the current from the cyclotron would be very much less than in the internal source case. Fairly good currents of heavier elements easily obtained in a gas are achieved in Japan (Mi 70). Yoshitoshi Miyazawa's group has developed a dual cathode (tungsten), water cooled anode source that operates inside the cyclotron at an arc power of 2.4 kW. (The cathodes erode at 0.3 grams per hour and limit the life to 20 hrs.) Extracted beams of C$^{4+}$ (4 μA), N$^{5+}$ (1 μA), and O$^{5+}$ (0.4 μA) have been obtained. An attempt to get a good Ne$^{5+}$ beam was unsuccessful. The external source at Saclay has provided beams of Ne and also higher charge states of the other elements, viz. Ne$^{7+}$, C$^{5+}$, and O$^{6+}$, 500 nA of each out of the source (Kn 69). The beam out of the cyclotron would be weak for these ions. Now and in the foreseeable future, any type of polarized source would have to be located outside the cyclotron, of course. Negative ion sources of high current output tend to be large and have high power requirements.
The Cyclotron Corporation, for instance, has very success-
fully used an axial injector in this application (Cl 69b),
as described previously. The system described in the pres-
ent feasibility study is not suitable for negative ion
injection because of the difficulty of holding large posi-
tive potentials on the electrodes. Also the space charge
limit has not been evaluated.

It thus appears that as a general principle, if a
given type of ion source can be constructed to operate in
the center of the cyclotron, superior results would be ob-
tained by doing so; external injection is needed for the
types of sources which cannot be squeezed into the cyclo-
tron.
6. Conclusion

The method of injecting ions from an external source into a sectored cyclotron by cycloid-like motion along the sector edge, which was first used by Gladyshev's group in Moscow on a model cyclotron using an injection energy 8.6% of the final energy (Gl 65), can also be used in a full scale application. The system studied in this paper would provide performance estimated to be roughly comparable to that of the other types of injectors now in use at cyclotrons of around the size as the one at MSU. The injection energy of 0.3% of the final energy is the maximum that will allow the injected beam to be bent into a suitable orbit near the cyclotron center with attainable electric field strengths. A buncher can be used to increase the transmission efficiency although a lack of isochronism in the injection path puts limits on its usefulness. Ions of different energies end up at different points near the machine center thus causing an effective increase
in horizontal emittance area. The optical properties during drift along the sector edge are almost linear, but not while in the central inflector system. This necessitates making some questionable approximations. A reasonably large vertical acceptance of the system can be obtained by a carefully optimized set of electrodes in this region. The effective horizontal emittance is considerably less than some other injection systems but for high quality present day polarized ion sources the total transmission efficiency is estimated to be comparable. There is essentially no depolarization in the cyclotron magnetic field and fringe field. External sources of highly charged heavier ions could be used but the current out of the cyclotron would be very low. The injection system investigated here is not appropriate for negative ions. With a 2% duty factor the energy spread in the beam is about 0.2% which should give high extraction efficiency.
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